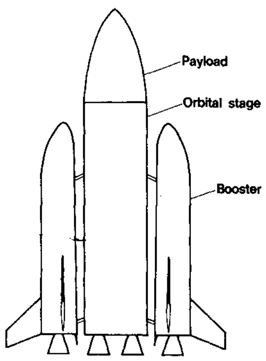


**Fig. 2 Companion system sketch.**

Mach 3.0 and glide back to the launch site. The masses given are based on using no crossfeed of propellant from the boosters to the orbital stage. Crossfeed could increase the payload capability of the Companion system, but the difference is probably not worth the complexity. The masses shown in Table 1 are based on numerical integration of a trajectory into a low polar orbit.

The rocket engines used for both the booster and the orbital stage of the Companion system use kerosene and oxygen propellants. Hydrogen is used for cooling and power generation. The results shown are based on using equal thrust levels on all three elements.

This analysis indicated that the Companion system could deliver a payload and shroud mass of about 10,000 kg, with a gross mass of about 250,000 kg. This capability should allow the Companion system to compete very effectively wih expendable launch vehicle and provide a useful augmentation to the Space Shuttle.

## Conclusion

Analysis of a Companion launch vehicle operating in conjuction with the Space Shuttle indicates that a vehicle could deliver 10,000 kg payloads into low-Earth orbit.

# Thermal Network Correction by the Optimization Method

K. Badari Narayana* and H.N. Murthy†
*ISRO Satellite Centre, Bangalore, India*

## Nomenclature

$A$ = area, $m$
$C$ = conduction exchange factor, W/K
$F$ = objective function
$K$ = number of load cases
$N$ = number of nodes
$n$ = number of parameters
$P$ = power dissipated, W
$R$ = radiation exchange factor, $W/K^4$
$T$ = temperature, °C
$x$ = parameter
$y$ = variable
$\alpha$ = absorptivity
$\epsilon$ = emissivity

*Superscripts*
0 = initial
$U$ = upper limit
* = dimensionless

*Subscripts*
cal = calculated
L = lower limit
obs = measured

Received Aug. 3, 1985; revision received Jan. 17, 1986. 

*Engineer, Thermal Systems Division.
†Head, Thermal Systems Division.

## Introduction

TEMPERATURE predictions of a satellite and its subsystems using the widely used lumped parameter system are subject to uncertain parameter values. Whenever there is a large deviation between measured and predicted temperature values, the parameter values have to be corrected. The trial-and-error method of correcting, though the simplest, becomes cumbersome as the model size increases. A good review of the available correction methods is given by Ishimoto and Pan.[1] The sequential estimation scheme[2,3] requires a smaller matrix inversion compared to one-pass methods, but an exact knowledge of measurement and system noise is needed. However, this information is usually not available. Regression[4] and singular-value decomposition[5] methods may estimate negative values for some parameters and require temperature values of all nodes. Both these methods yield satisfactory results only in case of low noise data.[6] Doenecke[7] has presented an iteration method for adjusting a thermal mathematical model by minimizing the sum of the squares of the residuals of the nodal heat-balance equations. Selection of an optical coating pattern that minimizes temperature excursions from some preselected temperatures is given by Costello et al.[8] using optimization methods.

The purpose of this note is to present the simplex optimization technique for satellite thermal network correction. The concept is simple for computer application and does not have the disadvantages of the other correction methods.

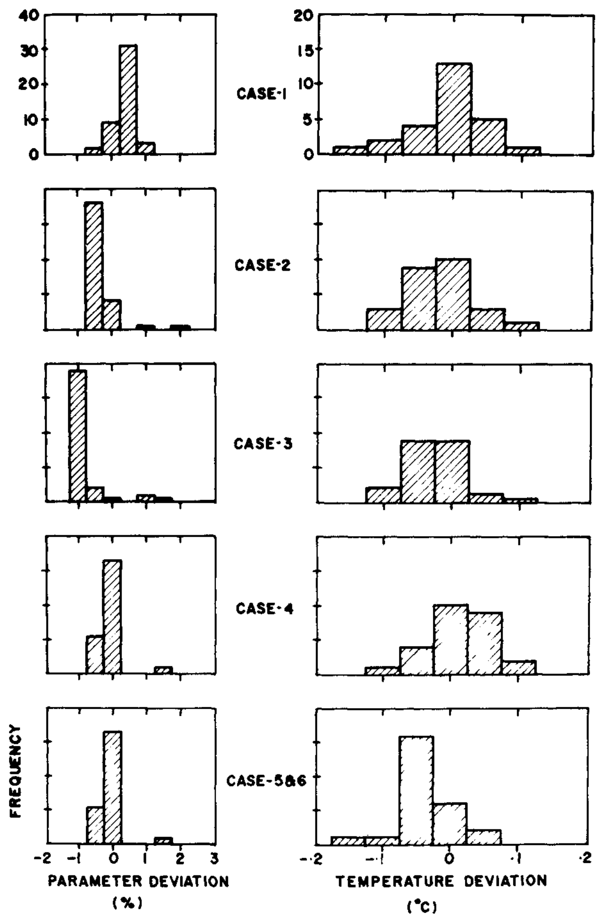


**Fig. 1 Histograms of parameter and temperature deviation.**

Table 1 True and corrected parameter values

| Node no. | | Conduction-exchange factor | | Node no. | | Radiation-exchange factor | |
|---|---|---|---|---|---|---|---|
| From | To | True | Corrected | From | To | True | Corrected |
| 1 | 2 | 0.217 | 0.21706 | 1 | 2 | 112.7 | 113.18 |
| 1 | 13 | 1.304 | 1.51874 | 1 | 11 | 48.7 | 48.94 |
| 2 | 3 | 0.446 | 0.44599 | 1 | 13 | 80.1 | 80.45 |
| 3 | 4 | 0.351 | 0.35097 | 2 | 3 | 306.3 | 307.75 |
| 3 | 5 | 0.001 | 0.00101 | 2 | 4 | 228.7 | 229.73 |
| 3 | 6 | 0.002 | 0.00202 | 2 | 8 | 364.1 | 365.92 |
| 3 | 8 | 0.063 | 0.06378 | 2 | 9 | 1221.2 | 1227.28 |
| 3 | 9 | 0.039 | 0.03899 | 3 | 4 | 453.4 | 455.85 |
| 4 | 5 | 0.095 | 0.09455 | 3 | 8 | 1159.3 | 1164.79 |
| 4 | 6 | 0.012 | 0.01194 | 3 | 9 | 126.7 | 127.30 |
| 4 | 7 | 0.027 | 0.02700 | 4 | 5 | 124.9 | 125.49 |
| 4 | 8 | 0.408 | 0.40803 | 4 | 8 | 110.1 | 110.62 |
| 4 | 9 | 0.078 | 0.07799 | 5 | 6 | 13.5 | 13.56 |
| 4 | 11 | 0.001 | 0.00100 | 6 | 7 | 6.8 | 6.79 |
| 5 | 6 | 0.006 | 0.00603 | 6 | 8 | 46.7 | 46.92 |
| 5 | 7 | 0.001 | 0.00100 | 7 | 8 | 87.0 | 87.40 |
| 5 | 8 | 0.193 | 0.19391 | 7 | 9 | 194.4 | 195.10 |
| 5 | 9 | 0.003 | 0.00301 | 7 | 10 | 139.1 | 139.78 |
| 6 | 8 | 0.025 | 0.02512 | 8 | 9 | 302.4 | 303.83 |
| 7 | 8 | 0.006 | 0.00603 | 9 | 10 | 268.3 | 269.57 |
| 8 | 9 | 0.939 | 0.94394 | 9 | 11 | 19.3 | 19.42 |
| 8 | 10 | 0.002 | 0.00201 | 10 | 11 | 12.9 | 13.02 |
| 9 | 11 | 0.068 | 0.06832 | | | | |
| 12 | 13 | 0.006 | 0.00603 | | | | |

Table 2 $\chi^2$ evaluation

| | $\chi^2$ value | |
|---|---|---|
| Case | Dimensional objective function | Normalized objective function |
| 1 | 0.15649 | 0.00614 |
| 2 | 0.03567 | 0.00614 |
| 3 | 0.32049 | 0.02304 |
| 4 | 0.02051 | 0.02309 |
| 5 | 0.02305 | 0.00553 |
| 6 | 0.02305 | 0.00553 |

## Analysis

The basis of the method is the evaluation of the parameters, which minimizes the objective function given as

$$F=\sum_{j=1}^{K}\sum_{i=1}^{N}(T_{\mathrm{cal}}-T_{\mathrm{obs}})^2 \tag{1}$$

with constraints

$$x_{i,L}\le x_i\le x_i^U \qquad i=1,\ldots n \tag{2}$$

Many optimization schemes are available in the literature[9] which can be used for minimizing Eq. (1). In the present investigation, the simplex scheme[10] is used. The constrained optimization problem is converted into an unconstrained problem by the application of the following transform:

$$x_i=x_{i,L}+(x_i^U-x_{i,L})\sin^2 y_i \qquad i=1,\ldots n \tag{3}$$

## Application

Parameter correction of the spin-stabalized RSD2 satellite is presented. The satellite consists of 13 isothermal nodes with 24 and 22 conduction and radiation-exchange factors, respectively. For purpose of this study, theoretically evaluated steady-state temperature values corresponding to sun aspect angles of 25 deg and 90 deg are considered, instead of the measured data. Twelve parameters (six each of conduction and radiation-exchange factors) are perturbed randomly—but within the limits of uncertainty—and are used as initial estimates. The simplex scheme is applied to reduce the initial objective function value (OFV), Eq. (1), to a prescribed small value.[11] To allow a uniform step length, all the parameters are normalized with respect to their initial estimates as

$$x_i^*=x_i/x_i^0 \qquad i=1,\ldots n \tag{4}$$

## Results

Corrections for all the 46 parameters are obtained since bound values are applied for all of them, even though only 12 of them are perturbed. The maximum temperature deviation due to perturbation is +7° and −3°C. The following cases are studied:

Case 1. Here 26 observed temperatures corresponding to the two load conditions are used. Table 1 shows the corrected parameter and the percentage difference from the true values. All the parameters are corrected except $C_{1\text{-}13}$, which is insensitive to temperature calculations.[12]

Case 2. In this case, only incomplete temperature data are available. In practice, temperature values of all the nodes may not be available due to insufficient numbers of temperature sensors, wrong mounting or snapping of sensors, etc. To simulate this situation, the temperature of nodes 1 and 11, and 4 and 9 for the two load conditions are not considered for OFV evaluation. Nodes 1, 4 and 11, 9 correspond to external and internal nodes, respectively.

The result of the correction is observed to be good, as shown in Fig. 1. The merits of the method compared to the regression method are that a large deviation of a particular conduction-exchange factor does not result in large deviation in the corresponding radiation-exchange factor and vice versa, no negative values for parameters are estimated, and temperatures of all nodes are not required.

Case 3. Thirteen temperature values corresponding to 90-deg sun-aspect angle only are used.

Case 4. Same as case 3, except that temperatures of nodes 4 and 9 are not considered.

Case 5. Thirteen temperature values correponding to 25-deg sun-aspect angle only are used.

Case 6. Same as case 5, except that temperature of nodes 4 and 10 are not considered.

Figure 1 shows the good matching obtained in all the cases.

The optimization procedure continues as long as there is a reduction in OFV. However, due to bad scaling of the OFV, the reduction need not always result in better parameter

**Table 3 Initial, perturbed, and corrected parameter values**

| Sl. No. | Details | True value | Perturbed value | Difference, % | Uncertainty, % | Corrected value | Corrected value[a] |
|---|---|---|---|---|---|---|---|
| 1. | $\epsilon_1$[b] | 0.8 | 0.75 | −6.25 | | 0.818 | 0.793 |
| 2. | $\epsilon_3$ | 0.819 | 0.85 | 3.79 | ±10.0 | 0.803 | 0.803 |
| 3. | $\epsilon_{12}$ | 0.7 | 0.67 | −4.29 | | 0.698 | 0.705 |
| 4. | $A_4$ | 2733 | 2800 | 2.45 | | 2818.0 | 2722.4 |
| 5. | $A_5$ | 570 | 555 | −2.63 | ±5.0 | 557.8 | 580.3 |
| 6. | $A_6$ | 132 | 130 | −1.52 | | 130.8 | 133.5 |
| 7. | $P_{10}$ | 0.7 | 0.65 | −7.14 | | 0.658 | 0.700 |
| 8. | $P_{11}$ | 1.0 | 0.93 | −7.00 | | 0.943 | 0.982 |
| 9. | $\alpha_2$ | 0.666 | 0.7 | 5.11 | ±10.0 | 0.672 | 0.673 |
| 10. | $\alpha_3$ | 0.881 | 0.8 | −9.19 | | 0.873 | 0.854 |
| 11. | $C_{4,7}$ | 0.027 | 0.024 | −11.11 | | 0.026 | 0.026 |
| 12. | $C_{4,8}$ | 0.408 | 0.450 | 10.29 | ±50.0 | 0.477 | 0.430 |
| 13. | $C_{9,11}$ | 0.068 | 0.060 | −11.76 | | 0.062 | 0.073 |
| 14. | $R_{2,8}$ | 364.1 | 400 | 9.86 | | 496.5 | 441.5 |
| 15. | $R_{2,9}$ | 1221.2 | 1000 | −18.11 | ±25.0 | 1241.2 | 1103.7 |
| 16. | $R_{7,9}$ | 194.4 | 150 | −22.84 | | 184.1 | 165.5 |

[a]Corresponds to rounded-off temperature values to nearest 0.5°C. [b]Subscripts correspond to node numbers.

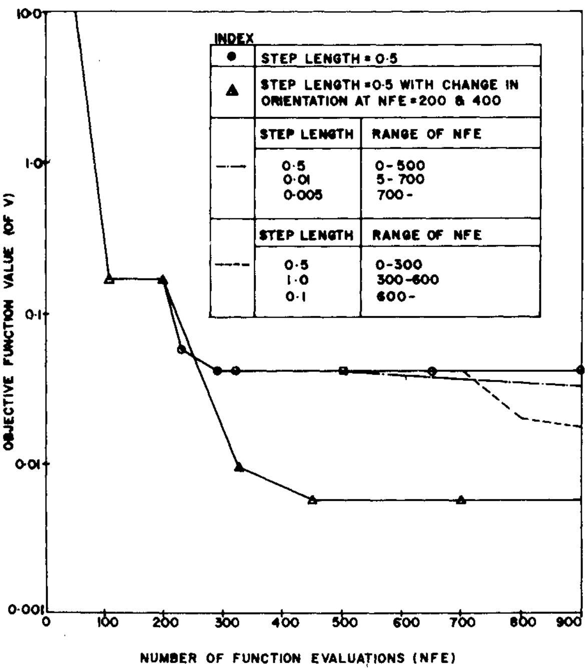


Fig. 2 Objective function value vs number of function evaluations.

values.[13] In such cases, to provide equal weighting to all the parameters, normalized OFV is used and is defined as

$$F = \sum_{j=1}^{K} \sum_{i=1}^{N} [(T_{\text{cal}} - T_{\text{obs}})/T_{\text{obs}}]^2 \tag{5}$$

The $\chi^2$ value for the fit between observed and measured states for dimensional and normalized OFV cases is shown in Table 2. It is preferable to use as many load conditions data (temperature measurements) as possible in a single calculation to obtain corrected parameter values.

There is no method available to choose an initial orientation or step length for the simplex to give the best extremum value of the function. However, after a few trials, one can locate the stage where the simplex is to be reoriented or new set of vertices considered. Figure 2 shows the rate of improvement in optimization due to change in orientation or step length.

The optimization method considered here for conduction and radiation exchange factors can be used for correcting the other thermal parameters, namely, radiation properties, power dissipation, etc. Table 3 shows the result for such parameters.

Both external and internal node parameters are corrected to a large extent. Large deviations in some parameters are due to the insensitivity of the parameter to temperature. Further improvement in parameter values is found to take more computational time. Correction results for observed temperature values rounded off to the nearest 0.5°C are also indicated. It is always possible to correct only those large parameters, keeping the other corrected parameter values constant.

## References

[1]Ishimoto, T. and Pan, H.M., "Thermal Network Correction Techniques," *Progress in Astronautics and Aeronautics,* Vol. 24: *Heat Transfer and Spacecraft Thermal Control,* edited by J.W. Lucas, AIAA, 1970, pp. 383-409.

[2]Browne Jr., B.H., "Statistical Estimation and Elimination of Thermal Network Model Errors," *Transactions of the ASME,* Ser. C, Vol. 91, 1969, pp. 554-560.

[3]Ishimoto, T. et al., "Development of Digital Computer Program for Thermal Network Correction," *Final Report, Phase I—Investigation/Feasibility Study,* TRW Systems, Redondo Beach, CA, Rep-11027-6001-RP-00, 1969.

[4]Shimoji, S. and Oshima, K., "A New Thermal Network Correction Method by the Statistical Regression Analysis," *Progress in Astronautics and Aeronautics,* Vol. 65: *Thermophysics and Thermal Control,* edited by R. Viskanta, AIAA, 1979, pp. 344-365.

[5]Shoda, K. and Yoneda, K., "Statistical Identification of Satellites Heat Transfer Characteristics," AIAA Paper 79-1104, June 1979.

[6]Shimoji, S., "A Comparison of Thermal Network Correction Methods," AIAA Paper 81-1139, June 1981.

[7]Doenecke, J., "Adjustment of a Thermal Mathematical Model to Test Data," *Journal of Spacecraft and Rockets,* Vol. 7, June 1970, pp. 720-726.

[8]Costello, F.A. et al., "Coating Selection Program," NASA CR 1041, 1968.

[9]Himmelblau, D.M., *Applied Nonlinear Programming,* McGraw Hill, New York, 1972.

[10]Nelder, J.A. and Mead, R., "A Simplex Method for Function Minimization," *Computer Journal,* Vol. 7, 1964, pp. 308-313.

[11]Badari Narayana, K., "Thermal Network Correction by Optimization Method," Mechanical Systems Division, ISRO Satellite Centre, Bangalore, India, Doc. ISAC-22-83-07-05-10, July 1983.

[12]Badari Narayana, K., "Uncertainty and Sensitivity Analysis in Satellite Thermal Design," Mechanical Systems Division, ISRO Satellite Centre, Bangalore, India, Doc. ISAC-22-83-08-05-05, Aug. 1983.

[13]Badari Narayana, K., "Studies on Thermal Network Correction," Mechanical Systems Division, ISRO Satellite Centre, Bangalore, India, Doc. ISAC-22-83-12-05-010, Dec. 1983.

# Errata

## Prediction of Burning Rates in Nozzleless Rocket Motors

Merrill K. King
*Atlantic Research Corporation*
*Alexandria, Virginia*

[J. Spacecraft, 22, 394-395 (1985)]

THE equations for $A_1$ and $A_2$ [Eqs. (7) and (8)] are reversed, and the bracketed term following $P$ in Eq. (8) should have been superscripted. The corrected equations should read as follows:

$$A_1 = k_3 V^{k_4} P^{[k_5 + k_6 \ln(V)]} \tag{7}$$

$$A_2 = 1 - k_1 V^{-k_2} \tag{8}$$

This equation reversal also occurs in AIAA Paper 82-1200, on which this Synoptic is based.